

Name: _____

Instructor: _____

Math 10560, Exam 1
February 18, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
.....					
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
.....					
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
.....					
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

Total _____

Name: _____

Instructor: _____

Multiple Choice

1.(6 pts) The function

$$f(x) = x^3 + x + \ln(x)$$

is a one-to-one function (there is no need to check this). What is $(f^{-1})'(2)$?

- (a) $\frac{1}{5}$ (b) 5 (c) $\frac{2}{27}$ (d) $\frac{27}{2}$ (e) $\frac{1}{e^5}$

2.(6 pts) Find the derivative of the function

$$f(x) = \frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}}.$$

(Logarithmic differentiation might help.)

- (a) $\frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[\frac{5}{x^2 - 1} + \frac{2}{x^2 + x + 1} - \frac{1}{2(x^2 + 1)} \right]$
- (b) $\frac{10x}{x^2 - 1} + \frac{4x + 2}{x^2 + x + 1} - \frac{x}{x^2 + 1}$
- (c) $\frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[\frac{5 \ln(x^2 - 1)}{x^2 - 1} + \frac{2 \ln(x^2 + x + 1)}{x^2 + x + 1} - \frac{\ln(x^2 + 1)}{x^2 + 1} \right]$
- (d) $\frac{(x^2 - 1)^5(x^2 + x + 1)^2}{\sqrt{x^2 + 1}} \left[\frac{10x}{x^2 - 1} + \frac{4x + 2}{x^2 + x + 1} - \frac{x}{x^2 + 1} \right]$
- (e) $\frac{5 \ln(x^2 - 1)}{x^2 - 1} + \frac{2 \ln(x^2 + x + 1)}{x^2 + x + 1} - \frac{\ln(x^2 + 1)}{x^2 + 1}$

Name: _____

Instructor: _____

3.(6 pts) Compute the integral

$$\int_0^{\log_3 5} \frac{3^x}{1+3^x} dx.$$

(a) $\ln 3(\ln 6 - \ln 2)$

(b) $3(\ln 6 - \ln 2)$

(c) $\frac{1}{\ln 3}(\ln 6 - \ln 2)$

(d) $\frac{1}{3}(\ln 6 - \ln 2)$

(e) $\ln 6 - \ln 2$

4.(6 pts) Compute the integral

$$\int_1^e \frac{1}{x(1+(\ln x)^2)} dx.$$

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2} - 1$

(c) $\frac{\pi}{4} - 1$

(d) 1

(e) $\frac{\pi}{2}$

Name: _____

Instructor: _____

5.(6 pts) Simplify the function

$$\cos(\sin^{-1}\left(\frac{x}{2}\right)).$$

(a) $\frac{\sqrt{4-x^2}}{x}$

(b) $\frac{x}{\sqrt{4-x^2}}$

(c) $\sqrt{4-x^2}$

(d) $\frac{\sqrt{4-x^2}}{2}$

(e) $\frac{\sqrt{4-x^2}}{4}$

6.(6 pts) Evaluate the limit

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x^2}.$$

(a) 0

(b) $-\infty$

(c) $-\frac{1}{2}$

(d) $+\infty$

(e) $\frac{1}{2}$

Name: _____

Instructor: _____

7.(6 pts) Evaluate the integral

$$\int_0^1 x e^{2x} dx.$$

- (a) $\frac{e^2 - 1}{4}$ (b) $\frac{e^2 + 1}{2}$ (c) $\frac{e^2 + 1}{2}$ (d) 1 (e) $\frac{e^2 + 1}{4}$

8.(6 pts) Evaluate the integral

$$\int \sin(5x) \cos(3x) dx.$$

Note: One of the formulas given on the last page of the exam may help you with this problem.

- (a) $-\frac{1}{2} \left[\frac{\cos(5x)}{5} + \frac{\cos(3x)}{3} \right] + C$ (b) $\frac{1}{2} \left[\frac{\sin(2x)}{2} - \frac{\sin(8x)}{8} \right] + C$
(c) $-\frac{1}{2} \left[\frac{\cos(2x)}{2} - \frac{\cos(8x)}{8} \right] + C$ (d) $\frac{1}{2} \left[\frac{\sin(2x)}{2} + \frac{\sin(8x)}{8} \right] + C$
(e) $-\frac{1}{2} \left[\frac{\cos(2x)}{2} + \frac{\cos(8x)}{8} \right] + C$

Name: _____

Instructor: _____

9.(6 pts) Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \tan^{100} x \sec^4 x \, dx.$$

- (a) 2 (b) $\frac{1}{101} + \frac{1}{103}$ (c) $\frac{1}{100} + \frac{1}{101}$
(d) $\frac{1}{100} - \frac{1}{101}$ (e) $\frac{1}{101} - \frac{1}{103}$

10.(6 pts) Which of the following expressions gives the correct form of the partial fraction decomposition of the function f shown below?

$$f(x) = \frac{3x^2 + 2x + 1}{(x - 1)(x - 4)^2(x^2 + 1)^2}$$

- (a) $\frac{A}{x - 1} + \frac{B}{(x - 4)^2} + \frac{Cx + D}{(x^2 + 1)^2}$
(b) $\frac{A}{x - 1} + \frac{B}{x - 4} + \frac{C}{(x - 4)^2} + \frac{D}{x^2 + 1} + \frac{E}{(x^2 + 1)^2}$
(c) $\frac{A}{x - 1} + \frac{B}{x - 4} + \frac{C}{(x - 4)^2} + \frac{Dx + E}{x^2 + 1} + \frac{Fx + G}{(x^2 + 1)^2}$
(d) $\frac{A}{x - 1} + \frac{Bx + C}{(x - 4)^2} + \frac{Dx + E}{(x^2 + 1)^2}$
(e) $\frac{A}{x - 1} + \frac{B}{(x - 4)^2} + \frac{C}{(x^2 + 1)^2}$

Name: _____

Instructor: _____

Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (10 pts.) let $M(t)$ denote the amount of a chemical substance remaining after t years where the initial amount is given by $M(0)$. The rate of decay of the substance is such that 40% of the initial amount is left after 10 years. It is known that the substance decreases at a rate proportional to the amount present a time t , that is $C'(t) = kC(t)$ for some constant k .

(a) What is the value of k ?

(b) What is the half-life of this substance (what is the amount of time it takes to decay to 50% of its original size)?

Name: _____

Instructor: _____

12. (15 pts.) Compute the integral

$$\int \frac{x^2 + 3x}{(x - 2)(x^2 + 2x + 2)} dx .$$

Name: _____

Instructor: _____

13. (15 pts.) Calculate the integral

$$\int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx .$$

Note: One of the formulas given on the last page of the exam may help you with this problem.

Name: _____

Instructor: _____

The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

Name: _____

Instructor: ANSWERS

Math 10560, Exam 1
February 18, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(●)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(●)	(e)
.....					
3.	(a)	(b)	(●)	(d)	(e)
4.	(●)	(b)	(c)	(d)	(e)
.....					
5.	(a)	(b)	(c)	(●)	(e)
6.	(a)	(b)	(c)	(●)	(e)
.....					
7.	(a)	(b)	(c)	(d)	(●)
8.	(a)	(b)	(c)	(d)	(●)
.....					
9.	(a)	(●)	(c)	(d)	(e)
10.	(a)	(b)	(●)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

11. _____

12. _____

13. _____

Total _____